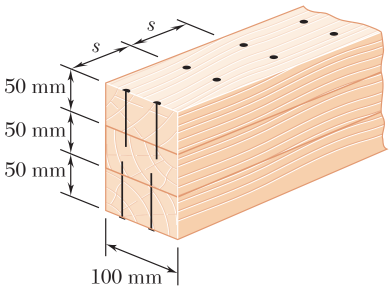


PROBLEM 6.1

Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing *s* that can be used between each pair of nails.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$

$$= 28.125 \times 10^{-6} \text{ m}^4$$

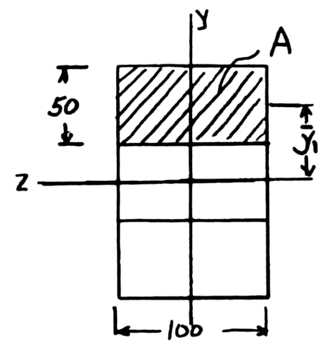
$$A = (100)(50) = 5000 \text{ mm}^2$$

$$\overline{y}_1 = 50 \text{ mm}$$

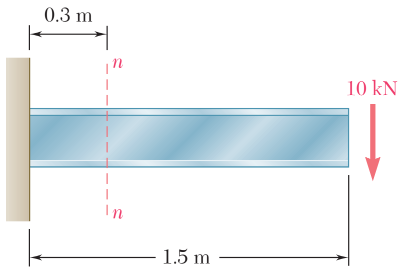
$$Q = A\overline{y}_1 = 250 \times 10^3 \text{ mm}^3 = 250 \times 10^{-6} \text{ m}^3$$

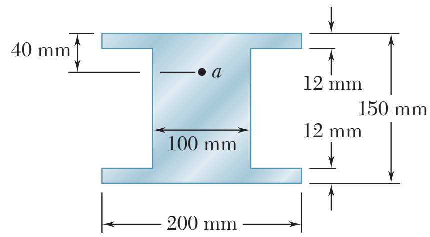
$$q = \frac{VQ}{I} = \frac{(1500)(250 \times 10^{-6})}{28.125 \times 10^{-6}} = 13.3333 \times 10^3 \text{ N/m}$$

$$qs = 2F_{\text{nail}} \qquad s = \frac{2F_{\text{nail}}}{q} = \frac{(2)(400)}{13.3333 \times 10^3} = 60.0 \times 10^{-3} \text{ m}$$



$s = 60.0$ mm ◀





PROBLEM 6.10

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION

At section *n-n*, $V = 10$ kN.

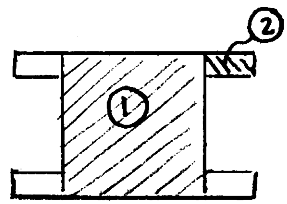
$$I = I_1 + 4I_2$$

$$= \frac{1}{12}b_1h_1^3 + 4\left(\frac{1}{12}b_2h_2^3 + A_2d_2^2\right)$$

$$= \frac{1}{12}(100)(150)^3 + 4\left[\left(\frac{1}{12}\right)(50)(12)^3 + (50)(12)(69)^2\right]$$

$$= 28.125\times10^6 + 4\left[0.0072\times10^6 + 2.8566\times10^6\right]$$

$$= 39.58\times10^6\ \text{mm}^4 = 39.58\times10^{-6}\ \text{m}^4$$



(*a*)

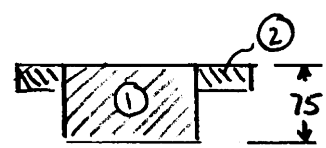
$$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$

$$= (100)(75)(37.5) + (2)(50)(12)(69)$$

$$= 364.05\times10^3\ \text{mm}^3 = 364.05\times10^{-6}\ \text{m}^3$$

$$t = 100\ \text{mm} = 0.100\ \text{m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10\times10^3)(364.05\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 920\times10^3\ \text{Pa}$$



$\tau_{max} = 920$ kPa ◀

(*b*)

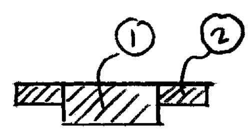
$$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$

$$= (100)(40)(55) + (2)(50)(12)(69)$$

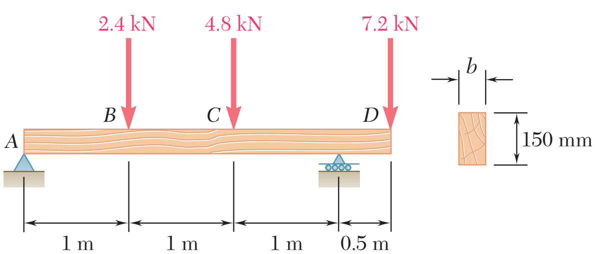
$$= 302.8\times10^3\ \text{mm}^3 = 302.8\times10^{-6}\ \text{m}^3$$

$$t = 100\ \text{mm} = 0.100\ \text{m}$$

$$\tau_a = \frac{VQ}{It} = \frac{(10\times10^3)(302.8\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 765\times10^3\ \text{Pa}$$



$\tau_a = 765$ kPa ◀



PROBLEM 6.18

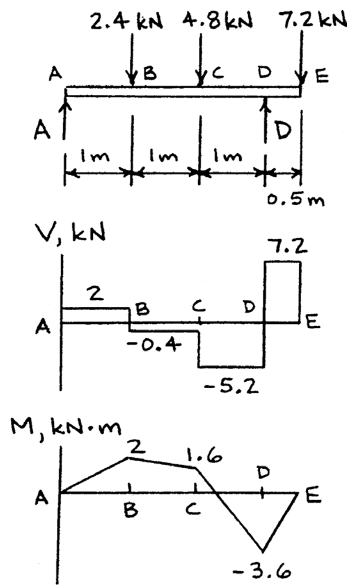
For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, $\sigma_{\text{all}} = 12$ MPa and $\tau_{\text{all}} = 825$ kPa.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \; -3A + (2)(2.4) + (1)(4.8) - (0.5)(7.2) = 0$$

$$\mathbf{A} = 2 \text{ kN} \uparrow$$

Draw the shear and bending moment diagrams.



$$|V|_{\max} = 7.2 \text{ kN} = 7.2 \times 10^3 \text{ N}$$

$$|M|_{\max} = 3.6 \text{ kN}\cdot\text{m} = 3.6 \times 10^3 \text{ N}\cdot\text{m}$$

Bending:

$$\sigma = \frac{M}{S}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma} = \frac{3.6 \times 10^3}{12 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$

For a rectangular section,

$$S = \frac{1}{6} b h^2$$

$$b = \frac{6S}{h^2} = \frac{(6)(300 \times 10^3)}{(150)^2} = 80 \text{ mm}$$

Shear: Maximum shearing stress occurs at the neutral axis of bending for a rectangular section.

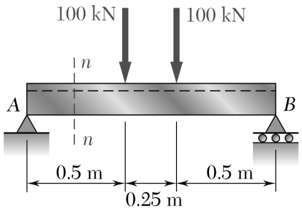
$$A = \frac{1}{2} bh, \quad \bar{y} = \frac{1}{4} h, \quad Q = A\bar{y} = \frac{1}{8} b h^2$$

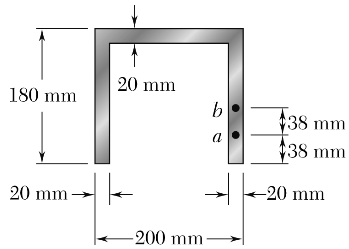
$$I = \frac{1}{12} b h^3 \qquad t = b$$

$$\tau = \frac{VQ}{It} = \frac{V(\frac{1}{8} b h^2)}{(\frac{1}{12} b h^3)(b)} = \frac{3}{2} \frac{V}{bh}$$

$$b = \frac{3V}{2h\tau} = \frac{(3)(7.2 \times 10^3)}{(2)(150 \times 10^{-3})(825 \times 10^3)} = 87.3 \times 10^{-3} \text{ m}$$

The required value of b is the larger one. $b = 87.3$ mm ◀





PROBLEM 6.21

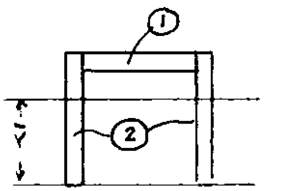
For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) the shearing stress at point *b*.

SOLUTION

$$R_A = R_B = 100 \text{ kN}$$

At section *n-n*, $V = 100 \text{ kN}$

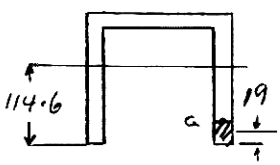
Locate centroid and calculate moment of inertia.



Part	$A(\text{mm}^2)$	$\overline{y}(\text{mm})$	$A\overline{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\overline{I}(\text{mm}^4)$
①	3200	170	544000	55.4	9821312	106667
②	7200	90	648000	24.6	4357152	19440000
Σ	10400		1192000		14178464	19546667

$$\overline{Y} = \frac{\Sigma A\overline{y}}{\Sigma A} = \frac{1192000}{104.00} = 114.6 \text{ mm}$$

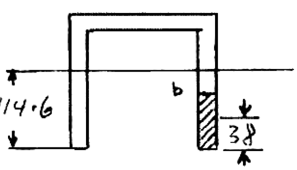
$$I = \Sigma Ad^2 + \Sigma \overline{I} = 14178464 + 19546667 = 33.725 \times 10^6$$



(*a*) $Q_a = A\overline{y} = (20)(38)(114.6 - 19) = 72656 \text{ mm}^3$

$t = 20 \text{ mm}$

$$\tau_a = \frac{VQ}{It} = \frac{(100000)(72656)}{(33.725 \times 10^6)(20)} = 10.8 \text{ MPa}$$ ◀



(*b*) $Q_b = A\overline{y} = (20)(76)(114.6 - 38) = 1164.32 \text{ mm}^9$

$t = 20 \text{ mm}$

$$\tau_a = \frac{VQ}{It} = \frac{(100000)(116432)}{(33.725 \times 10^6)(20)} = 17.3 \text{ MPa}$$ ◀